to an intermediate one and disappears entirely in the free-molecular mode. This is confirmed by a comparative analysis of experimental data on isothermal Poiseuille flow and thermomolecular pressure.

NOTATION

Here N_{Kn} is the Knudsen number; γ, the universal exponent of thermomolecular pressure; R_0, the radius of a cylindrical capillary; P, the pressure; T, the temperature, ν, the logarithmic pressure gradient; τ, the logarithmic temperature gradient; U, the macroscopic gas velocity; q, the thermal flux density; R, the gas rarefaction index; l, the length of the mean free path; I_N, the numerical mean-over-the-section gas flux; I_q, the mean-over-the-section thermal flux; and α, the tangential-momentum accommodation coefficient.

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MEASUREMENT OF NONSTATIONARY HEAT FLUXES BY "AUXILIARY WALL" SENSORS

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UDC 536.24.083

Computational dependences are obtained to determine the nonstationary heat flux by using sensors executing the method of an auxiliary wall. The dependences are valid for an arbitrary relationship between the thermophysical properties of the sensor and the object on which it is located.

The peculiarities of measuring nonstationary heat fluxes by heat meters executing the method of an auxiliary wall are considered in [1]. A number of dependences is presented to determine the flux $q(\tau)$ of heat meters located on the surface of a semi-infinite body for particular values of the thermophysical properties of the heat meter and the base, defined by the magnitude of the criterion $\varkappa = (\lambda_2/\lambda_1)\sqrt{(a_1/a_2)} = 0;\ 1.0;\ \infty$. A solution of the problem is presented below for any values of $\varkappa$. As in [1], the model of the heat meter is represented in the form of a plate located on a half-space (sketch). The temperature fields of the heat meter $t_1(x, \tau)$ and the base $t_2(x, \tau)$ are described by the equations

$$\frac{\partial t_i}{\partial \tau} = a_i\left(\frac{\partial^2 t_i}{\partial x^2}\right),\ i = 1;\ 2. \tag{1}$$

Leningrad Institute of Precision Mechanics and Optics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 1, pp. 99-103, July, 1979. Original article submitted July 4, 1978.

0022-0841/79/3701-0824$07.50

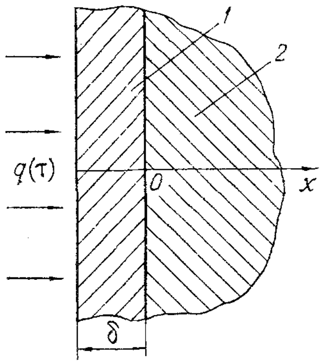

Fig. 1. Model of a heat meter on a half-space.

The surface x = $-\delta$ absorbs the heat flux

$$q(\tau)=-\lambda_1 \frac{\partial t_1}{\partial x}\Big|_{x=-\delta}, \quad (2)$$

whose magnitude must be determined. Other boundary conditions have the following form:

$$\frac{\partial t_2}{\partial x}\Big|_{x\to\infty}=0, \text{ or } t_2|_{x\to\infty}=\text{const}, \quad (3)$$

$$\lambda_1 \frac{\partial t_1}{\partial x}\Big|_{x=0}=\lambda_2 \frac{\partial t_2}{\partial x}\Big|_{x=0}, \ t_1|_{x=0}=t_2|_{x=0},$$

$$t_i|_{\tau=0}=t_w, \ i=1;\ 2.$$

Let us note that the following are assumed in formulating and solving the problem: the thermophysical properties are independent of the temperature and an ideal thermal contact holds between bodies 1 and 2.

The solution of the problem formulated in the form of transforms can be written as [1]

$$\Delta\theta(s)=Y_q(s)\,Q(s), \quad (4)$$

$$Y_q(s)=\frac{\sqrt{a_1}}{\lambda_1}\,\frac{\operatorname{ch}\sqrt{\frac{s}{a_1}}\,\delta+\varkappa\operatorname{sh}\sqrt{\frac{s}{a_1}}\,\delta-1}{\sqrt{s}\left(\operatorname{sh}\sqrt{\frac{s}{a_1}}\,\delta+\varkappa\operatorname{ch}\sqrt{\frac{s}{a_1}}\,\delta\right)}, \quad (5)$$

from which the transform of the desired flux is

$$Q(s)=\frac{1}{Y_q(s)}\cdot\Delta\theta(s). \quad (6)$$

Analyzing (6) it is easy to show that $1/Y_q(s)$ is of the order of $\sqrt{s}$ and hence, additional manipulations presented in [1] are necessary.

Consequently, it is established that the form of the function $\varphi(\xi)$ which is the original of the expression

$$F(s)=\frac{1}{sY_q(s)}=\frac{\lambda_1}{\sqrt{a_1}}\,\frac{\operatorname{sh}A\sqrt{s}+\varkappa\operatorname{ch}A\sqrt{s}}{\sqrt{s}\,(\varkappa\operatorname{sh}A\sqrt{s}+\operatorname{ch}A\sqrt{s}-1)}, \quad (7)$$

$$A=\frac{\delta}{\sqrt{a_1}}, \quad \varkappa=\frac{\lambda_2}{\lambda_1}\sqrt{\frac{a_1}{a_2}},$$

must be found to solve the problem.

Furthermore, to determine the flux, the dependence can be used [1]:

$$q(\tau)=\int_0^\tau \varphi(\tau-\xi)\,\frac{d\,[\Delta t(\xi)]}{d\xi}\,d\xi. \quad (8)$$

The derivative of the temperature drop, which can result in substantial errors in a practical determination from tests in a number of cases, is in the integrand. It is hence expedient to convert the dependence (8) so as to avoid the derivative $d[\Delta t]/d\xi$. To do this we integrate by parts

$$q(\tau)=\varphi(\tau)\,\Delta t(\tau)-\int_0^{\tau}[\Delta t(\xi)-\Delta t(\tau)]\,\frac{d\varphi(\tau-\xi)}{d\xi}\,d\xi. \qquad (8a)$$

In contrast to [1], expressions are obtained here for the function $\varphi(\xi)$ for arbitrary values of the parameter $\varkappa$ in the general case.

Let us find the original $\varphi(\xi)$ for (7); to do this we represent F(s) as the sum of two components

$$F(s)=\frac{1}{sY_q(s)}=\frac{\lambda_1}{\sqrt{a_1}}\,\frac{\operatorname{sh}A\sqrt{s}}{\sqrt{s}(\varkappa\operatorname{sh}A\sqrt{s}+\operatorname{ch}A\sqrt{s}-1)}+\frac{\lambda_1}{\sqrt{a_1}}\,\frac{\varkappa\operatorname{ch}A\sqrt{s}}{\sqrt{s}(\varkappa\operatorname{sh}A\sqrt{s}+\operatorname{ch}A\sqrt{s}-1)}. \qquad (9)$$

After expanding the values of the hyperbolic functions and simple manipulation, the first component can be written in the form

$$\frac{\lambda_1}{\sqrt{a_1}}\,\frac{\operatorname{sh}A\sqrt{s}}{\sqrt{s}(\varkappa\operatorname{sh}A\sqrt{s}+\operatorname{ch}A\sqrt{s}-1)}=\frac{\lambda_1}{\sqrt{a_1 s}}\,\frac{1}{\varkappa+1}\,\frac{1+\exp(-A\sqrt{s})}{1+\dfrac{\varkappa-1}{\varkappa+1}\exp(-A\sqrt{s})}. \qquad (10)$$

Let us use the expansion [2]

$$\frac{1}{1+m\exp(-A\sqrt{s})}\sum_{n=0}^{\infty}(-1)^n\exp(-nA\sqrt{s})\,m^n; \qquad (11)$$

$$m=\frac{\varkappa-1}{\varkappa+1}$$

and put (10) in the form

$$\frac{\lambda_1}{\sqrt{a_1}}\,\frac{\operatorname{sh}A\sqrt{s}}{\sqrt{s}(\varkappa\operatorname{sh}A\sqrt{s}+\operatorname{ch}A\sqrt{s}-1)}=\frac{\lambda_1}{\sqrt{a_1}}\,\frac{1}{\varkappa+1}\,\frac{1+\exp(-A\sqrt{s})}{\sqrt{s}}\sum_{n=0}^{\infty}(-1)^n\exp(-nA\sqrt{s})\left(\frac{\varkappa-1}{\varkappa+1}\right)^n. \qquad (12)$$

Proceeding analogously with the second component in (9), we obtain

$$\frac{\lambda_1}{\sqrt{a_1 s}}\,\frac{\varkappa\operatorname{ch}A\sqrt{s}}{(\varkappa\operatorname{sh}A\sqrt{s}+\operatorname{ch}A\sqrt{s}-1)}=\frac{\lambda_1}{\sqrt{a_1 s}}\,\frac{\varkappa}{(\varkappa+1)}$$

$$\times\left[2\exp(-A\sqrt{s})\sum_{n=0}^{\infty}\exp(-nA\sqrt{s})\sum_{n=0}^{\infty}(-1)^n\exp(-nA\sqrt{s})\right. \qquad (13)$$

$$\left.\times\left(\frac{\varkappa-1}{\varkappa+1}\right)^n+[1-\exp(-A\sqrt{s})]\sum_{n=0}^{\infty}(-1)^n\exp(-nA\sqrt{s})\left(\frac{\varkappa-1}{\varkappa+1}\right)^n\right].$$

It can be shown that the product of the sums in the right side of (13) is reduced to the form [5]

$$\sum_{n=0}^{\infty}\exp(-nA\sqrt{s})\sum_{n=0}^{\infty}(-1)^n\exp(-nA\sqrt{s})\left(\frac{\varkappa-1}{\varkappa+1}\right)^n=\sum_{n=0}^{\infty}\left[\exp(-nA\sqrt{s})\sum_{k=0}^{n}(-1)^k\left(\frac{\varkappa-1}{\varkappa+1}\right)^k\right]. \qquad (14)$$

Substituting the values of (12) and (13) into (9) and performing simple manipulations, we obtain

$$F(s)=\frac{\lambda_1}{\sqrt{a_1 s}}\,\frac{1}{(\varkappa+1)}\sum_{n=0}^{\infty}\left\{\left[1+\exp(-A\sqrt{s})+\varkappa[1\right.\right.$$

$$\left.-\exp(-A\sqrt{s})]\right](-1)^n\exp(-nA\sqrt{s})\left(\frac{\varkappa-1}{\varkappa+1}\right)^n \qquad (15)$$

$$\left.+2\varkappa\exp(-A\sqrt{s})\exp(-nA\sqrt{s})\sum_{k=0}^{n}(-1)^k\left(\frac{\varkappa-1}{\varkappa+1}\right)^k\right\}.$$

We find the original of the function F(s) after performing an inverse Laplace transformation. We consequently obtain

$$L^{-1}[F(s)] = \varphi(\tau) = \frac{\lambda_1}{\sqrt{\pi a_1 \tau}(\varkappa+1)} \sum_{n=0}^{\infty} \left\{ (1+\varkappa)(-1)^n \right.$$
$$\times \exp\left(-\frac{n^2A^2}{4\tau}\right)\left(\frac{\varkappa-1}{\varkappa+1}\right)^n + \left[(-1)^n(1-\varkappa)\left(\frac{\varkappa-1}{\varkappa+1}\right)^n\right. \tag{16}$$
$$\left.\left. + 2\varkappa \sum_{k=0}^{n} (-1)^k \left(\frac{\varkappa-1}{\varkappa+1}\right)^k\right] \exp\left[-\frac{(n+1)^2A^2}{4\tau}\right]\right\}.$$

Let us substitute the value of $\varphi(\tau)$ into (8a) and perform the necessary manipulations, and after simplification we obtain a computational dependence to determine the flux desired

$$q(\tau) = \frac{\lambda_1 \Delta t(\tau)}{\sqrt{\pi a_1 \tau}(\varkappa+1)} \sum_{n=0}^{\infty} \left\{ (-1)^n(\varkappa+1)\left(\frac{\varkappa-1}{\varkappa+1}\right)^n \exp\left(-\frac{n^2A^2}{4\tau}\right) \right.$$
$$+ \left[(-1)^n(1-\varkappa)\left(\frac{\varkappa-1}{\varkappa+1}\right)^n + 2\varkappa\sum_{k=0}^{n}(-1)^k\left(\frac{\varkappa-1}{\varkappa+1}\right)^k\right]$$
$$\left. \times \exp\left[-\frac{(n+1)^2A^2}{4\tau}\right]\right\} - \frac{\lambda_1}{4\sqrt{\pi a_1}(\varkappa+1)} \int_0^{\tau} [\Delta t(\xi) - \Delta t(\tau)]$$
$$\times \frac{1}{\sqrt{(\tau-\xi)^5}} \sum_{n=0}^{\infty} \left\{(-1)^n(1+\varkappa)\left(\frac{\varkappa-1}{\varkappa+1}\right)^n [n^2A^2 - 2(\tau-\xi)]\right.$$
$$\times \exp\left[-\frac{n^2A^2}{4(\tau-\xi)}\right] + \left[(-1)^n(1-\varkappa)\left(\frac{\varkappa-1}{\varkappa+1}\right)^n\right.$$
$$\left.\left. + 2\varkappa\sum_{k=0}^{n}(-1)^k\left(\frac{\varkappa-1}{\varkappa+1}\right)^k\right][(n+1)^2A^2 - 2(\tau-\xi)] \exp\left[-\frac{(n+1)^2A^2}{4(\tau-\xi)}\right]\right\} d\xi. \tag{17}$$

Verification of (17) for passages to the limit ($\varkappa = 1$, $\varkappa = 0, \infty$) resulted in the computational relationships obtained in [1].

To estimate the applicability of the dependence (17) to compute the nonstationary flux $q(\tau)$, a program was compiled for the ES 1020 electronic computer, and the results of experiments performed on the test stand whose construction is described in [3] were processed. Let us note that the test stand permits both giving nonstationary fluxes varying according to different laws known to the experimenter, and measuring them by using different heat meters. The experimental test-stand investigations exhibited good reproducibility of the results (the error does not exceed 5%).

An "auxiliary wall" heat meter of the type DTP-02, developed in the Institute of Engineering Thermophysics of the Ukrainian of Science [4], was used as sensor. The heat meter was located on a bulky copper plate ($\varkappa \approx 35$). The heat flux incident on the sensor varied linearly between 0 and $5 \cdot 10^2$ W/m^2 at the rates $dq/d\tau$ = 20-35 W/m^2·sec. It was established as a result of the investigations that the maximum discrepancy between the fluxes given by using the test stand and those measured by the heat meter with a subsequent computation by means of (17) does not exceed 8%. The computation time on the ES 1020 electronic computer does not exceed several minutes.

Therefore, to measure the nonstationary heat fluxes by using "auxiliary wall" heat meters fastened to a semi-infinite body, the thermophysical properties of the heat meter and the semi-infinite body ($\varkappa \approx (\lambda_2/\lambda_1)\sqrt{a_1/a_2}$) as well as the temperature drop at different times, must be known.

NOTATION

λ_1, a_1, thermal conductivity and thermal diffusivity of the heat meter; λ_2, a_2, the same for the base; $q(\tau)$, heat flux density; τ, time; $t_1(x, \tau)$, $t_2(x, \tau)$, temperature of the heat meter and the base; δ, thickness of the heat meter; x, running coordinate; $\Delta t(\tau)$, temperature drop over the heat meter thickness; $Q(s)$, $\Delta\theta(s)$, Laplace transforms of the heat flux $q(\tau)$ and the temperature drop $\Delta t(\tau)$; s, Laplace transform parameter; $Y_q(s)$ transfer function from the heat flux to the temperature drop.

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DISPERSION OF THE DIELECTRIC PERMITTIVITY IN MIXTURES OF NEMATIC LIQUID CRYSTALS

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UDC 532.783:548.0:537.226

The spectrum of relaxation time for the low-frequency dispersion has been found to become wider in mixtures of nematic liquid crystals, which can be interpreted as a superposition of two Debye relaxation mechanisms.

The dielectric properties of uniformly oriented nematic liquid crystals are determined in terms of two principal values of the dielectric permittivity $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$, corresponding to measurements respectively along and across the axis of nematic order [1].

In single-component nematic liquid crystals at superhigh frequencies (hundreds of megahertz) $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ have a range of Debye dispersion with relaxation times ($\tau_{\parallel}$ and $\tau_{\perp}$ respectively) close to those in an isotropic liquid (τ_{is}) [2]. At low frequencies (hundreds of kilohertz) $\varepsilon_{\parallel}$ has an additional range of dispersion [3]. One associates the low-frequency range of dispersion with rotation of molecules about their short axes, this rotation being greatly inhibited by the nematic order, and the high-frequency range of dispersion with rotation of molecules about their long axes. The latter rotation differs insignificantly little from analogous rotation in the isotropic phase.

In a multicomponent system of nematic liquid crystals the pattern of dipole relaxation can be much more complex.

The object of this study was to experimentally analyze the dispersion of the dielectric permittivity in mixtures of nematic liquid crystals.

The compounds dealt with in this study are listed in Table 1.

Experimental. The dielectric characteristics were measured over the 0.1-10 MHz frequency range by the resonance method with a set of "Tangens-2M" instruments. The test cell consisted of a capacitor with plane-parallel silver plates with the gap between them not exceeding 2.5 mm. The capacitance with air between the plates was 3.2 pF. The measurement error did not exceed 0.6 and 7% for ε' and ε'' respectively. A uniform orientation of the nematic liquid crystals was achieved by means of a constant magnetic field of 6 kG strength. The temperature was maintained accurately within ±0.1°C. More details about the experimental apparatus and procedure can be found in the earlier reports [4,5].

Results. The frequency dependence of the dielectric characteristics ($\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$) was determined from measurements at discrete frequencies 0.1, 0.2, 0.5, 1.1, 2, 5, 10 MHz. Their temperature dependence was studied in the mesophase and in the isotropic-liquid phase.

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0022-0841/79/3701-0828$07.50